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Unstable magnetic fluxes in heterotic string theory

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Abstract: We study exact string backgrounds representing a constant magnetic field background in heterotic string theory. These backgrounds are obtained by Kaluza-Klein reduction of a special class of plane wave solutions. For small values of the magnetic field they possess localized closed string tachyons analogous to the Nielsen-Olesen instability of a constant magnetic field in SO(3) Yang-Mills theory. When the magnetic field is embedded in the SO(32) gauge group of the heterotic string it is possible to study the lowest level tachyon in supergravity. We identify the closed string tachyons as fluctuations of the supergravity fields about the background. We argue that the tachyon signals the decay of the background to flat space. Our evidence rests on the study of the closed string tachyon potential, world sheet renormalization group equations in the supergravity approximation and the S-dual of this system in type-I theory. In the S-dual description, the closed string tachyons in heterotic string theory correspond to open string tachyons in type-I theory. Finally we analyze the non-perturbative stability of these models representing constant magnetic field backgrounds and show that it is stable to pair production of particles.

Keywords: Superstrings and Heterotic Strings, String Duality, Tachyon Condensation.

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Contents

1. Introduction

Open string tachyon condensation has been studied in a great detail following the work of Sen [1]. Open string tachyons occur in various non-supersymmetric configurations of D-branes. The tachyons are relevant operators on the boundary of the open string world sheet which modify the boundary conditions of the open string. The end point of tachyon condensation for the various systems studied is flat space or a supersymmetric brane configuration. For example the end point of condensation of the tachyonic mode when a D-brane and anti D-brane are brought on top of each other is flat space. In fact the minimum of the tachyon potential equals the tension of the D-branes.

On the other hand tachyons in closed string theory are relevant operators of the bulk world sheet theory. Condensation of these modes can potentially change the space time in which the string propagates. For example, it is well known that relevant operators in the bulk conformal field theory change its central charge [2]. At present we lack tractable methods to study bulk tachyons. There has been progress in understanding condensation of localized closed string tachyons [3]–[9]. These tachyons occur in various non-supersymmetric orbifolds in the twisted sector. The flow induced by these operators has been studied by various methods. It has been seen that condensation of these tachyons resolves the orbifold and the system flows to flat space.

Localized closed string tachyons are similar to open string tachyons, they are localized on defects in the space time, just as open string tachyons are localized on D-branes. It would be interesting to make this connection precise, by finding an example of a system in which the tachyonic mode can have both an open and closed string description. The existence of an open string dual also suggests that the closed string tachyon potential can be evaluated. In this paper we study such a system. These systems were first discovered by [10, 11] and studied in detail by [12, 13]. They are backgrounds representing uniform magnetic fields in string theory.

To motivate these backgrounds we first discuss a well known instability in Yang-Mills theory. It is known that a constant magnetic field in SO(3) Yang-Mills theory is unstable [14]. This instability is called the Nielsen-Olesen instability, there is an infinite degeneracy of tachyons at the lowest Landau level. The tachyon potential can be evaluated and it can be shown that the value of the potential at the minimum cancels the energy of the magnetic flux. Thus these tachyons signal the instability of the constant magnetic field to decay to the vacuum.

There are two known ways to embed magnetic fields in string theory. They are the Melvin backgrounds [15] and backgrounds obtained by Kaluza-Klein reduction of a special class of plane wave solutions found by [10] called chiral null models. In this paper we will study the latter. They are exact conformal field theories to all orders in α'. If the magnetic field is uniform they can be quantized in the light cone gauge [12, 13] When these models are embedded in type-II string theories they are supersymmetric. In heterotic string theory there are embeddings which have tachyons whose $(\text{mass})^2$ agrees with the Nielsen-Olesen instability for weak magnetic fields. There are two distinct situations which are related by a T-duality; (1) the magnetic field arises from a Kaluza-Klein gauge field, (2) the magnetic field is present in the SO(32) gauge group[1] of the heterotic string theory. To study tachyon condensation it is convenient to examine the latter. We identify the tachyon corresponding to the Nielsen-Olesen instability as a supergravity fluctuation. We argue that the tachyon condensation in this background drives it to flat space. We present three arguments in favour of this.

Firstly we discuss the S-dual of this background in type-I theory. In the type-I background closed string tachyons of the heterotic string background appear in the open string sector. We derive a decoupling limit in the type-I background in which the back reaction

[1]We choose to discuss the SO(32) heterotic string theory for definiteness, a similar discussion can be done in the $E_8 \times E_8$ heterotic string.

to the geometry can be neglected. In the decoupling limit the type-I open string can be quantized and the open string tachyon corresponds to the Nielsen-Olesen instability. Furthermore, in the decoupling limit the process of tachyon condensation is identical to the condensation of the Nielsen-Olesen tachyon in field theory. The tachyon drives the system to the Yang-Mills vacuum. Thus, turning on the coupling, by continuity we would expect that the tachyon in the heterotic string also drives the system to the vacuum. Next we analyze the tachyon condensation process directly in the heterotic string. We construct an energy functional and derive the closed string tachyon potential. We show that the minimum of the tachyon potential cancels the background magnetic field and the system is driven to flat space. In the various non-supersymmetric orbifolds studied so far there has been only an indirect evaluation of the closed string tachyon potential in [7]. This model admits a direct evaluation of the tachyon potential. Finally we study the world sheet renormalization group flow in supergravity. We show that it is possible to obtain a consistent set of flow equations for small magnetic fields. It is shown that the tachyon drives the RG flow to flat space.

Background fields in a theory also can decay by pair production of particles. For example constant electric fields decay by pair production of electrons and positrons [16], the Melvin background contains a magnetic field which can decay by pair production of Kaluza-Klein monopoles [17, 18, 19]. It is of interest to know if the backgrounds studied in this paper are unstable to pair production of particles. For this it is convenient to study the case when the magnetic field in heterotic string theory arises from a Kaluza-Klein gauge field. By examining the corrections to the effective action describing vacuum polarization we show the amplitude for pair production of particles vanishes.

The organization of this paper is as follows. In section 2 we review the Nielsen-Olesen instability and the evaluation of the tachyon potential. In section 3 we introduce the plane wave solutions which represent constant magnetic field backgrounds in string theory. Section 4 introduces the background in heterotic string theory in which the magnetic field arises from the SO(32) gauge group. We identify the tachyons as supergravity fluctuations. In section 5 we argue that tachyon condensation in this background drives it to flat space. In section 6 we show that constant magnetic fields in heterotic string theory are stable to decay by pair production of particles.

2. The Nielsen-Olesen instability

In this section we briefly review the Nielsen-Olesen instability in non-abelian gauge theories [14]. We first discuss the linearized fluctuations around a constant magnetic field in SO(3) Yang-Mills theory in $3+1$ dimensions and show that there are unstable modes in the lowest Landau level. We choose the gauge group to be SO(3) as it can be embedded trivially in the SO(32) gauge group of the heterotic string. Then we evaluate the tachyon potential and show that the instability corresponds to the decay of the magnetic field to the Yang-Mills vacuum.

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2.1 Fluctuations around a constant magnetic field

The action is given by of SO(3) Yang-Mills theory in $3+1$ dimensions is given by

$$S_{\mathrm{YM}} = -\frac{1}{4g_{\mathrm{YM}}^2}\int d^4x\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\,. \tag{2.1}$$

Here the SO(3) gauge group generators are given by following 3×3 anti-symmetric matrices

$$L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \qquad L_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.2}$$

They obey the usual SO(3) commutation relations $[L_i, L_j] = i\epsilon_{ijk}L_k$. We choose the constant magnetic field to be along the 3 direction and in the 3 direction of the gauge group. The field configuration in the Landau gauge is given by

$$A_0 = 0\,, \qquad A_1 = Qfx^2L_3\,, \qquad A_2 = 0\,, \qquad A_4 = 0\,. \tag{2.3}$$

Without loss of generality we can take $Qf > 0$.[2]

We now now analyze the fluctuations around this background and show that there are unstable modes in the lowest Landau level. A general fluctuation can be written as

$$\delta A_\mu = W_\mu^+ L_+ + W_\mu^- L_- + \phi_\mu L_3\,, \tag{2.4}$$

where $L_+ = L_1 + iL_2$ and $L_- = L_1 - iL_2$. The linearized equation of motion obeyed by the fluctuation is given by

$$D^\mu D_\mu \delta A_\nu - 2i[F^\mu{}_\nu, \delta A_\mu] = 0\,, \tag{2.5}$$

where D_μ refers to the covariant derivative given by $D_\mu = \partial_\mu + i[A_\mu,\ ]$. A_μ and $F_{\mu\nu}$ are the background gauge fields. In the above equation we have imposed the background gauge condition

$$D^\mu \delta A_\mu = 0\,. \tag{2.6}$$

Consider the off diagonal fluctuation W_μ^+ with $W_0^+ = W_3^+ = 0$. The linearized equation of motion for this fluctuation reduces to

$$\left(\partial^0\partial_0 + \partial^3\partial_3 + (\partial_1 - ifQx^2)^2 + \partial_2^2\right)W_a^+ - 2i\epsilon_{ab}fQW_b^+ = 0\,, \tag{2.7}$$

here $a, b = 1, 2$ and $\epsilon_{12} = -\epsilon_{21} = 1$. It is now clear that the eigenvalue problem reduces to the familiar Landau level problem in quantum mechanics. From (2.7) it can be seen that we can label the eigen functions by energy, E and the momentum along 1 and 3 directions which we call k_1 and k_3 respectively. Using these eigen values we obtain the following eigen value equation for the energy

$$\left(E^2 - k_3^2 - (k_1 - fQx^2)^2 + \partial_2^2\right)W_a^+ - 2i\epsilon_{ab}fQW_b^+ = 0\,. \tag{2.8}$$

[2]For gauge fields in SO(32) heterotic string theory $Q = \sqrt{2}$.

The energy eigen values are given by

$$E^2 = k_3^2 + 2fQ(l + 1/2) \mp 2fQ\,. \tag{2.9}$$

The last term arises from the fact that $i\epsilon_{ab}$ acts as a Pauli matrix σ^2 with eigen values ± 1, l labels the Landau level. It is now easy to see that for $k_3 = 0, l = 0$ the energy is complex which signals a tachyon, the wave function of this tachyon is given by.

$$W_a^+(k_1) = \left(\frac{fQ}{4\pi l_1^2}\right)^{1/4} e^{-ik_1x^1} e^{-\frac{Qf}{2}(-\frac{k_1}{Qf}+x^2)^2} \begin{pmatrix} 1 \\ i \end{pmatrix}, \tag{2.10}$$

$E^2 = -fQ$ for this wave function and it satisfies the background gauge condition. The wave function is a gaussian and therefore these tachyons are localized. We have normalized this wave function by requiring

$$\int dx^1 dx^2 (W^+)^\dagger(k_1) W^+(k_1') = \frac{2\pi}{l_1}\delta(k_1 - k_1')\,. \tag{2.11}$$

To regulate the system we confine it to a large box of size l_1 and l_2 in the x and y directions. Note that this energy is not a function of k_1 and therefore infinitely degenerate, the degeneracy is given by $fQl_1l_2/(2\pi)$. For completeness we discuss the tachyons from W_a^-, which obey the equation

$$\left(E^2 - k_3^2 - (k_1 + fQx^2)^2 + \partial_2^2\right) W_a^- + 2i\epsilon_{ab} fQ W_b^- = 0\,. \tag{2.12}$$

The energy eigen-values are given by $E^2 = k_3^2 + 2fQ(l+1/2) \pm 2fQ$. Thus again there is a tachyon in the lowest Landau level, the normalized tachyon wave function which satisfies the background gauge condition is given by

$$W_a^-(k_1) = \left(\frac{fQ}{4\pi l_1^2}\right)^{1/4} e^{-ik_1x^1} e^{-\frac{fQ}{2}(\frac{k_1}{fQ}+x^2)^2} \begin{pmatrix} 1 \\ -i \end{pmatrix}. \tag{2.13}$$

Note that the tachyon wave functions in (2.10) and (2.13) are related by $(W^+(k_1))^* = W^-(-k_1)$, this is true because the gauge field is hermitean. Higher Landau levels are obtained by multiplying the wave functions in (2.10) and (2.13) by the appropriate Hermite polynomial.

Finally we analyze the diagonal fluctuation ϕ_μ. These fluctuations commute with the background given in (2.3) therefore they are not charged and they obey the Klein-Gordan equation

$$\partial^\nu \partial_\nu \phi_\mu = 0\,. \tag{2.14}$$

The wave functions satisfying the background gauge condition are given by

$$\phi_a(k_1, k_2) = \frac{1}{\sqrt{l_1 l_2}} \frac{i\epsilon_{ab}k_b}{|k|} e^{ik_1x^1 + ik_2x^2}\,, \tag{2.15}$$

with $\phi_0 = \phi_3 = 0$ and $|k| = \sqrt{k_1^2 + k_2^2}$, we have set $k_3 = 0$. The i in the above equation is introduced so that $\phi_a^*(k) = \phi_a(-k)$. The normalization of these wave functions is fixed so that

$$\int dx^1 dx^2 \phi_a^*(k)\phi_a(k') = \frac{4\pi^2}{l_1 l_2}\delta^2(k - k')\,. \tag{2.16}$$

It is useful to state the form of the wave function for the off diagonal fluctuations in the gauge for which the background field is given by

$$A_0 = 0\,, \qquad A_i = \frac{Qf}{2}\epsilon_{ij}x^j L_3\,, \qquad A_4 = 0\,. \tag{2.17}$$

The lowest Landau level wave functions is a gaussian given by

$$W_a^+ = e^{-\frac{Qf}{2}\left((x^1)^2+(x^2)^2\right)}\begin{pmatrix}1\\ i\end{pmatrix}, \qquad W_a^- = (W_a^+)^*\,. \tag{2.18}$$

Higher Landau levels are obtained by multiplying the above wave function with the appropriate Laguerre polynomial.

2.2 Condensation of the Nielsen-Olesen tachyon

We now show that the Nielsen-Olesen instability signals the decay of the constant magnetic field in non-abelian gauge theories to the vacuum. We infer this using two methods. (1) We show that for a single unit of magnetic flux there is a tachyon potential such that the value at the minimum of the potential cancels the energy due to the magnetic flux. We follow the method developed in [20] for evaluating the tachyon potential for uniform magnetic fields on a 2-torus. We first restrict ourselves to the tachyonic modes in the lowest Landau level and the diagonal fluctuations. We expand the gauge field in these modes, substitute these expansions in the action (2.1) and eliminate the diagonal modes using the classical equations of motion to obtain the effective potential for the tachyon. We then argue that the property of the tachyon potential to cancel the energy of the magnetic flux will remain true even if the higher Landau levels are included. (2) We solve the equations of motion for small magnetic fields and show that there exists a solution with expectation values for the tachyon such that the field strength vanishes.

(i) The tachyon potential. We will show that tachyon wavefunctions with a gaussian profile in k_1 cancel a single unit of background flux. For the off diagonal fluctuation W^+, the tachyon wave function is given by

$$\begin{aligned} W^+(x^1,x^2) &= \left(\frac{fQ}{4\pi l_1^2}\right)^{1/4}\int \frac{dk_1 l_1}{2\pi}e^{-\frac{k_1^2}{2fQ}}W^+(k_1)\,,\\ &= \left(\frac{fQ}{4\pi l_1^2}\right)^{1/4}\frac{l_1}{2\pi}\sqrt{\pi f Q}\,e^{-\frac{fQ}{4}\left((x^1)^2+(x^2)^2-2ix^1x^2\right)}\begin{pmatrix}1\\ i\end{pmatrix}. \end{aligned} \tag{2.19}$$

Similarly the tachyon from the off diagonal fluctuation W^- with a gaussian profile is given by

$$W^-(x^1,x^2) = \left(\frac{fQ}{4\pi l_1^2}\right)^{1/4}\frac{l_1}{2\pi}\sqrt{\pi f Q}\,e^{-\frac{fQ}{4}\left((x^1)^2+(x^2)^2+2ix^1x^2\right)}\begin{pmatrix}1\\ -i\end{pmatrix}. \tag{2.20}$$

We allow an arbitrary profile $\phi(k)$ for the diagonal fluctuations, the expansion of the gauge field with these fluctuations is given by

$$A_a = A_a^{(0)} + \left(\frac{\pi l_2}{fQl_1}\right)^{1/4} \left(\chi W_a^+ L_+ + \chi^* W_a^- L_-\right) + \frac{l_1 l_2}{4\pi^2} \int d^2k \phi(k) \frac{i\epsilon_{ab} k_b}{\sqrt{l_1 l_2}|k|} e^{ik\cdot x} L_3 \,, \quad (2.21)$$

where χ stands for the expectation value of the tachyon and we have normalized χ for convenience. $A_a^{(0)}$ stands for background gauge field in (2.3), for a single unit of flux we have

$$\int F_{12} dx^1 \wedge dx^2 = 2\pi \,. \quad (2.22)$$

Therefore the field strength is given by $fQ = 2\pi/(l_1 l_2)$. Substituting the expansion (2.21) in the Yang-Mills action (2.1) and integrating out the diagonal fluctuations we obtain the following effective potential for the tachyon.

$$S = \frac{1}{4g_{\rm YM}^2} \int dx^0 dx^3 \left[\frac{16\pi^2}{l_1 l_2} + 16\left(-\frac{2\pi}{\sqrt{l_1 l_2}}|\chi|^2 + |\chi|^4\right)\right] . \quad (2.23)$$

We have provided the details of the evaluation of this potential in appendix A. Note that the action is a perfect square and that the minimum of the tachyon potential cancels the energy due to the background flux. Therefore a gaussian profile for the tachyon cancels a single unit of flux and the tachyon drives the system to the vacuum. Though the wave function is a gaussian it has support over the whole range of the box, this is the reason that the constant flux density is canceled by the tachyon profile. It will be interesting to find out the tachyon profile for larger units of flux

Now let us consider including the higher Landau levels in the analysis. The expectation values for the diagonal fluctuations $\phi(k)$ induces higher Landau levels to acquire expectation values. Integrating the higher Landau levels will introduce higher order terms in the effective action. Then the best way to analyze the effective potential of the tachyon is to look at the complete action given by

$$S = -\frac{1}{2g_{\rm YM^2}} \int dx^1 dx^2 \,{\rm Tr}(F_{12}F^{12}) \,. \quad (2.24)$$

This is the sum of squares, therefore the minimum is obtained at $F_{12} = 0$. Since there is no conserved charge as $\int dx^1 dx^2 \,{\rm Tr}(F_{12}) = 0$, there is no obstruction for the field strength to vanish.

(ii) Solution of equations of motion for weak fields. The equation of motion for the Yang-Mills action is given by $D^\mu F_{\mu\nu} = 0$. Consider the following expansion around the background (2.17)

$$A_a^3 = \frac{Qf}{2}\epsilon_{ab} x^b + \phi_a(x^1, x^2), \quad A_a^\pm = W_a^\pm(x^1, x^2) \,, \quad (2.25)$$

Let the diagonal components, including the background field f be of the $O(\epsilon^2)$ and the off diagonal components be of the $O(\epsilon)$. Let ϕ_a and $W_a^\pm$ satisfy the background gauge condition (2.6). We expand $W_a^\pm$ in eigen functions of the operator $O^\pm_{ab} = D^c D_c \delta_{ab} \mp 2i\epsilon_{ab} f Q$. The equation of motion in terms of ϕ_a and $W^\pm_{ab}$ to $O(\epsilon^3)$ are given by

$$\begin{aligned}
\partial_b F^3_{ba} &= 0 \\
\Big(O^+_{ab}W^+_b - iW^+_b\partial_b\phi_a + i\phi_b\partial_b W^+_a - iW^+_b(\partial_b\phi_a - \partial_a\phi_b) + & \\
+2W^+_b(W^+_b W^-_a - W^-_a W^+_b)\Big) &= 0 \\
\Big(O^-_{ab}W^-_b + iW^-_b\partial_b\phi_a - i\phi_b\partial_b W^-_a + iW^-_b(\partial_b\phi_a - \partial_a\phi_b) + & \\
+2W^-_b(W^-_b W^+_a - W^+_a W^-_b)\Big) &= 0\,. \qquad (2.26)
\end{aligned}$$

Here $F^3_{ab} = -Qf\epsilon_{ab} + \partial_a\phi_b - \partial_a\phi_b + 2i(W^+_a W^-_b - W^-_a W^+_b)$. We have also used the property that $F^\pm_{12} \sim O(\epsilon^3)$ in obtaining the above equations.

We now solve these equations and show that the solution corresponds to vanishing field strength. The eigenfunctions of the operator $O^\pm_{ab}$ always have a gaussian factor with a width $1/\sqrt{f}$, therefore derivatives of W_i are of order $O(\epsilon^3)$ and can be set to zero in the above set of equations. Then the equation of motion for ϕ_i is solved to $O(\epsilon^3)$ by $\phi_i = 0$. This makes it consistent to set coefficients of all the higher level Landau wave functions to zero and work only with the tachyon. The off-diagonal components obey the following relations

$$W^+_1 = iW^+_2\,, \quad W^-_i = (W^+_i)^*\,. \qquad (2.27)$$

Using these relations the equation of motion for W^+_1 reduces to

$$-QfW_1 + 4W_1|W_1|^2 = 0\,. \qquad (2.28)$$

Thus the solution is given by $|W^+_1|^2 = Qf/4$. Note that for this expectation value the field strength F^3_{ij} and $F^\pm_{ij}$ vanish. Furthermore, this expectation value is consistent with the fact the $W^\pm_i \sim O(\epsilon)$. Thus the tachyon condenses so as to cancel the background flux.[3]

3. Constant magnetic fields in string theory

In this section we present a brief review of exact string backgrounds representing constant magnetic fields in string theory. These form a special class of plane wave solutions found in [10]. There are embeddings of these plane wave solutions in heterotic string theory which have localized tachyons. For weak magnetic fields there exists a tachyon which can be identified with the Nielsen-Olesen tachyon. We show that this tachyon can be studied within supergravity if the magnetic field arises from the SO(32) gauge group of the heterotic string.

3.1 The bosonic string

Though the bosonic string has the usual bulk tachyon there are additional localized tachyons analogous to the Nielsen-Olesen tachyon in presence of magnetic fields. We will discuss the situation in detail for the case of the bosonic string and we will be brief for the super string theories, a detailed discussion can be found in [12, 13].

[3]The fact that the expectation value of the tachyon comes out to be constant is consistent with the fact that it is a gaussian since the gaussian has width $1/\sqrt{f}$ and it is almost constant for a large region.

We first introduce their supergravity description. Consider the following background in the bosonic string

$$
\begin{aligned}
ds^2 &= dudv + a_i dx^i du + dx^i dx_i + dx^m dx_m \,, \\
B_{iu} &= a_i \,, \qquad e^{2\Phi} = g_s^2 \,, \\
\text{with } a_i &= \epsilon_{ij}\frac{\tilde{f}}{2}x^j \,, \qquad \text{and} \qquad \tilde{f} = \sqrt{\frac{\alpha'}{2}} f \,.
\end{aligned} \tag{3.1}
$$

Here $i,j \in \{1,2\}$, $m,n \in \{3,4,\ldots,24\}$ and $u = \phi - t, v = \phi + t$. t is the time coordinate and ϕ is the last spatial coordinate, it is compactified on a circle of radius R. B_{iu} refers to the Neveu-Schwarz B field, the dilaton is constant. The solution in (3.1) has a null killing vector $\frac{\partial}{\partial v}$ and is a special class of the plane wave solutions found in [10]. On Kaluza-Klein reduction of this plane wave solution along ϕ one obtains a uniform magnetic field background in string theory. Using the Kaluza-Klein anstaz on (3.1) we obtain

$$
\begin{aligned}
ds^2 &= -(dt + a_i dx^i)^2 + dx^i dx_i + dx^m dx_m \\
A_i^{(1)} &= \frac{\alpha'}{R^2} a_i \qquad A_i^{(2)} = -\frac{\alpha'}{R^2} a_i \\
B_{ti} &= a_i \,, \qquad e^{2\Phi} = \frac{g_s^2\sqrt{\alpha'}}{R} \,.
\end{aligned} \tag{3.2}
$$

Here we have labelled the U(1) gauge fields according to the convention given in [21]. Now it is easy to see that the gauge potentials correspond to constant magnetic fields. This solution is different from the Melvin background [15] which also contains magnetic fields. The magnetic fields in the Melvin universe is non-uniform, the magnetic field strength decreases from a finite value at the origin to zero at infinity.

The world sheet action on the background given in (3.1) reduces to

$$
\begin{aligned}
S = -\frac{1}{4\pi\alpha'}\int d\sigma^+ d\sigma^- \Big(& \partial_+ u \partial_- v + \partial_+ v \partial_- u + \partial_+ X \partial_- \bar{X} + \partial_+ \bar{X} \partial_- X - \\
& - i\tilde{f}(X\partial_+\bar{X} - \bar{X}\partial_+ X)\partial_- u + 2\partial_+ x^m \partial_- x_m \Big) \,.
\end{aligned} \tag{3.3}
$$

Here $\sigma^+ = \sigma^0 + \sigma^1$ and $\sigma^- = -\sigma^0 + \sigma^1$. Note that the interaction terms proportional to $\tilde{f}$ is chiral. From this world sheet action it is easy to see that the plane wave representing a constant magnetic field background is exact to all orders in α'. Consider the following redefinitions

$$
X = e^{i\tilde{f}\tilde{u}(\sigma^-)} Y \,, \qquad \bar{X} = e^{-i\tilde{f}\tilde{u}(\sigma^-)} \bar{Y} \,, \tag{3.4}
$$

where $\tilde{u}(\sigma^-)$ refers to the right moving part of u. Then the action in (3.3) reduces to a free action in $u, v, Y, \bar{Y}, x^m$. We can quantize the world sheet action in (3.3) by fixing the light cone gauge so that

$$
u = u_0 + p\sigma^+ - \tilde{p}\sigma^- \,. \tag{3.5}
$$

Then as the coordinates X and $\bar{X}$ are single valued, the free coordinates $Y, \bar{Y}$ obey twisted boundary conditions.

$$
Y(\sigma^0, \sigma^1 + 2\pi) = e^{2\pi i\tilde{f}\tilde{p}} Y(\sigma^0, \sigma^1) \,, \qquad \bar{Y}(\sigma^0, \sigma^1 + 2\pi) = e^{-2\pi i\tilde{f}\tilde{p}} \bar{Y}(\sigma^0, \sigma^1) \,. \tag{3.6}
$$

With these boundary conditions these fields can be mode expanded as

$$Y = i\sqrt{\alpha'} \sum_{n=-\infty}^{\infty} \left(\frac{|n-\nu|^{1/2}}{n-\nu} a_n e^{-i(n-\nu)\sigma^+} + \frac{|n+\nu|^{1/2}}{n+\nu} \tilde{a}_n e^{i(n+\nu)\sigma^-} \right),$$
$$\bar{Y} = i\sqrt{\alpha'} \sum_{n=-\infty}^{\infty} \left(\frac{|n+\nu|^{1/2}}{n+\nu} b_n e^{-i(n+\nu)\sigma^+} + \frac{|n-\nu|^{1/2}}{n-\nu} \tilde{b}_n e^{i(n-\nu)\sigma^-} \right), \tag{3.7}$$

here $\nu = \tilde{f}\tilde{p}$ and with out loss of generality we can choose $0 < \nu < 1$. The commutation relations for the left moving oscillators are

$$[a_n, b_{-m}] = \delta_{n,m}, \qquad [b_n, a_{-m}] = \delta_{n,m}, \qquad \text{for } n, m = 1, 2, \ldots$$
$$\text{and } [b_0, a_0] = 1, \tag{3.8}$$

with similar commutation relations for the right movers. From these mode expansions and commutation relations it is easy to write down the spectrum of bosonic strings in the background (3.1). The spectrum for $k^m = 0$ is given by

$$M^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{wR}{\alpha'}\right)^2 + \frac{2}{\alpha'}\left(N + \tilde{N} - 2 + \nu(1-\nu)\right) +$$
$$+ 2(\tilde{M} + Q_R) f \left(l + \frac{1}{2}\right) - 2(\tilde{M} + Q_R) f S$$
$$\text{with } \tilde{N} - N = nw, \tag{3.9}$$

where n and w are the momentum and winding numbers along the compact direction ϕ, $\tilde{M}$ and Q, the right moving charge are given by

$$\tilde{M} = \sqrt{\frac{\alpha'}{2}} M, \qquad Q_R = \sqrt{\frac{\alpha'}{2}} \left(\frac{n}{R} - \frac{wR}{\alpha'}\right) \tag{3.10}$$

l stands for the occupation number of the zero mode left oscillators given by $a_0 b_0$, this is the Landau level. S is the angular momentum of the left moving oscillators in the $Y, \bar{Y}$ plane, for the bosonic string it is given by

$$S = \sum_{n=1}^{\infty} (b_{-n} a_n - a_{-n} b_n). \tag{3.11}$$

The mass formula in (3.9) is valid for $Q_R > 0$, the same formula with $Q_R \to -Q_R$ and $S \to -S$ is true for $Q_R < 0$.

We now show that there is a tachyon which corresponds to the Nielsen-Olesen tachyon in the bosonic string. Consider the state with $n = 1$, $w = -1$, $N = 1$, $\tilde{N} = 0$ at the self dual radius $R = \sqrt{\alpha'}$, then $S = \pm 1$ and the (mass)^2 of this state reduces to

$$M^2 = 2(\tilde{M} + Q) f \left(l + \frac{1}{2}\right) \mp 2(\tilde{M} + Q) f S + \frac{2}{\alpha'}\nu(1-\nu), \tag{3.12}$$

where $Q = \sqrt{2}$. Ignoring the zero point energy we see that for weak magnetic field $f \ll 4Q/\alpha'$ this state has the same mass formula as the Nielsen-Olesen tachyon in (2.9). In general when the compact direction is not at the self dual point the off diagonal gauge bosons charged under the background U(1) gauge field are massive. In fact the contribution to the $(\text{mass})^2$ in (3.9) from the winding and momentum modes are the masses of the off diagonal bosons. Note that the states which are tachyonic in general have both winding and momentum quantum numbers and higher spins, therefore they cannot be seen in supergravity. In superstring theories with constant magnetic field the $(\text{mass})^2$ formula basically remains the same with minor modifications. Superstring theories do not have the zero point energy making the identification with the Nielsen-Olesen tachyon more precise.

3.2 Type-II A/B string

The magnetic flux background in (3.1) can be embedded in either type-IIA or type-IIB string theory. The world sheet action can be obtained from the bosonic action in (3.3) by promoting the bosonic fields to super fields, it is given by

$$\begin{aligned} S &= S_{\text{Bosonic}} + S_{\text{Left}} + S_{\text{Right}} \\ S_{\text{Left}} &= -\frac{1}{4\pi\alpha'}\int d\sigma^+ d\sigma^- \left(i\partial_-\psi^v\psi^u + i\partial_-\psi^u\psi^v + i\partial_-\psi^X\psi^{\bar{X}} + i\partial_-\psi^{\bar{X}}\psi^X\right) + \\ &\qquad + 2i\partial_-\psi^m\psi_m + \tilde{f}\partial_- u\left(\psi^X\psi^{\bar{X}} - \psi^{\bar{X}}\psi^X\right) + \tilde{f}\partial_-\psi^u\left(X\psi^{\bar{X}} - \bar{X}\psi^X\right) \\ S_{\text{Right}} &= -\frac{1}{4\pi\alpha'}\int d\sigma^+ d\sigma^- \left(i\tilde{\psi}^v\partial_+\tilde{\psi}^u + i\tilde{\psi}^u\partial_+\tilde{\psi}^v + i\tilde{\psi}^X\partial_+\tilde{\psi}^{\bar{X}} + i\tilde{\psi}^{\bar{X}}\partial_+\tilde{\psi}^X\right) + \\ &\qquad + 2i\tilde{\psi}^m\partial_+\tilde{\psi}_m + \tilde{f}\tilde{\psi}^u\left(X\partial_+\tilde{\psi}^{\bar{X}} - \bar{X}\partial_+\tilde{\psi}^X + \tilde{\psi}^X\partial_+\bar{X} - \tilde{\psi}^{\bar{X}}\partial_+ X\right). \end{aligned} \tag{3.13}$$

Here the superscripts on the fermions label their super partners, S_{Bosonic} stands for the bosonic action in (3.3) and now m runs from $3, 4, \ldots, 8$. The compact direction ϕ is the 9th coordinate. Again this action is conformal to all orders in α' and it can be quantized in the light cone gauge.

The uniform magnetic field background in IIA/B string theory preserves 16 of the 32 supersymmetries [10, 22]. The Nielsen-Olesen type tachyons present in the bosonic string are projected out here by the GSO projection. As an example consider the same state we discussed for the case of the bosonic string with the quantum numbers $n = 1$, $w = -1$, $N = 1$, $\tilde{N} = 0$ in the Neveu-Schwarz sector at the self dual radius, this state corresponds to off diagonal gauge bosons. As this state has world sheet fermion number -1 for the left movers it is projected out by the GSO projection.

3.3 Heterotic string

The most symmetric way to embed the magnetic flux background in the heterotic string would be such that the action S_{Left} in (3.13) came from the left moving fermions and the S_{Right} from the fermions corresponding to the SO(32) gauge bosons. However the signature on the right moving fermions is lorentzian while the world sheet fermions corresponding to the gauge bosons in the heterotic string have Euclidean signature. Therefore this totally symmetric embedding is prohibited. We discuss the three ways the magnetic flux background (3.1) can be embedded in heterotic string theory. These are best described from the world sheet point of view.

(i) Left truncation. Let the fermions corresponding to the gauge bosons be left movers. Then the world sheet action for the left truncated embedding is obtained by dropping the action S_{Left}. The world sheet action is given by

$$S = S_{\text{Bosonic}} + S_{\text{Right}} - \frac{1}{2\pi\alpha'} \int d\sigma^+ d\sigma^- i\partial_- \lambda^A \lambda_A \,. \tag{3.14}$$

Here λ^A with $A = 1, \ldots, 32$ are the left moving fermions corresponding to the SO(32) gauge bosons. This embedding of the uniform magnetic field in heterotic string theory preserves 1/2 of the supersymmetries of the heterotic string [10]. In light cone gauge we can set $\tilde{\psi}^u = 0$. Therefore all terms proportional to the interaction $\tilde{f}$ drops out in S_{Right}. Then the quantization of the fermions is trivial. The off diagonal gauge bosons at the self dual radius which are charged under the background U(1) and which would have been the Nielsen-Olesen tachyons are projected out by the GSO projection. In this case the gauge bosons from the SO(32) gauge group is not charged under the background U(1) therefore they are also not tachyonic.

(ii) Right truncation. Now consider the fermions corresponding to the gauge bosons be right movers. The right truncated model is obtained by dropping the action S_{Right}. The world sheet action is given by

$$S = S_{\text{Bosonic}} + S_{\text{Left}} - \frac{1}{2\pi\alpha'} \int d\sigma^+ d\sigma^- i\tilde{\lambda}^A \partial_+ \tilde{\lambda}_A \tag{3.15}$$

This embedding of the uniform magnetic field in heterotic string is non-supersymmetric and there are tachyons. The world sheet action can be quantized in the light cone gauge, the mass spectrum in the Neveu-Schwarz sector is given by [13]

$$M^2 = Q_L^2 + 2f(Q_R + \tilde{M})\left(l + \frac{1}{2}\right) - 2f(Q_R + \tilde{M})S_L + \frac{4}{\alpha'}\left(N_L - \frac{1}{2}\right) \tag{3.16}$$

Here we have eliminated the level number of the right movers using the level matching condition. Q_L is the left moving charge given by

$$Q_L = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} + \frac{wR}{\alpha'}\right), \tag{3.17}$$

the left moving angular momentum S_L in the $Y, \bar{Y}$ plane has contribution both from the bosonic oscillators and the fermionic ones. It is easy to see that this model has Nielsen-Olesen type instabilities. Consider the states with $N_L = 1/2$, $Q_R = \sqrt{2}$, $Q_L = 0$ at the self dual radius where the gauge group is enhanced to SU(2), the (mass)^2 of this state is given by

$$M^2 = 2f(Q_R + \tilde{M})\left(l + \frac{1}{2}\right) \mp 2f(Q_R + \tilde{M}) \tag{3.18}$$

For small values of f this mass spectrum reduces to the spectrum of fluctuations of the off-diagonal gauge bosons in a SO(3) gauge theory with a uniform magnetic field. Therefore there is a Nielsen-Olesen instability of small f. In fact there is an infinite tower of tachyons from higher levels which are charged with respect to the background U(1). Consider the states with $N_L = 1/2+m^2$, $Q_R = m\sqrt{2}$, $Q_L = 0$, $l = 0$ at the self dual radius. For large m, $S_L \sim m^2$, these states are tachyons for $m > 2\sqrt{2}/(\alpha' f)$. Thus for arbitary small magnetic fields there are tachyons at higher levels with high spins [13]. All the tachyons have both winding modes and momentum modes or higher spins therefore they cannot be studied in supergravity. If on the other hand there exists an action for the enhanced gauge group coupled with gravity the tachyon in the lowest level, $N_L = 1/2$ can be studied. This is conveniently achieved by embedding the U(1) magnetic field in the SO(32) gauge group of the heterotic string.

(iii) Magnetic field from the SO(32) gauge group. Consider the Kaluza-Klein compactification of the right truncated model at the self dual radius. The supergravity solution is given in (3.2) with $R = \sqrt{\alpha'}$. We can obtain the supergravity solution with the magnetic field in the internal gauge group by applying a SO(1,17) T-duality transformation. This T-duality transforms the U(1) gauge fields arising from the Kaluza-Klein direction to a gauge field arising from the SO(32) gauge group, it leaves the metric and the B field invariant. We use the notation and formulae in [21]. At the self dual radius the moduli matrix M is a 18×18 identity matrix. The SO(1,17) transformation is given by

$$\Omega = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & I_{15} \end{pmatrix}. \tag{3.19}$$

Note that Ω satisfies $\Omega^T L \Omega = L$ with L given by

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -I_{16} \end{pmatrix} \tag{3.20}$$

here I_{15}, I_{16} are 15×15 and 16×16 dimensional identity matrices respectively. This SO(1,17) transformation basically exchanges the the right moving S^1 with a S^1 from the right moving internal torus. The gauge fields transform as $V_i \rightarrow \Omega V_i$ where V_i is a 18 dimensional vector. For the solution in (3.2) V_i is given by

$$V_i = \begin{pmatrix} A_i^{(1)} \\ A_i^{(2)} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{3.21}$$

ΩV_i has only one U(1) gauge field embedded in the SO(32) gauge group, given by $A_i^{\rm Int} = \sqrt{2} a_i$. The world sheet theory with the gauge field in the SO(32) gauge can also be quantized in the light cone gauge [13]. T-duality at the self dual radius is a gauge symmetry, therefore one can think of this T-duality also as a gauge transformation in the enhanced gauge group.[4] At the self dual point the gauge group is $\mathrm{SU}(2)_R \times \mathrm{SO}(32)$. The gauge field in the background (3.2) is embedded in the U(1) of $\mathrm{SU}(2)_R$. The background gauge field is given by

$$A_i^{(R)} = \frac{1}{\sqrt{2}}\left(A_i^{(1)} - A_i^{(2)}\right) = \sqrt{2}a_i \tag{3.22}$$

Now the gauge transformation which exchanges the $\mathrm{SU}(2)_R$ with an SU(2) of SO(32) embeds the background gauge field in the SO(32) gauge group.

Since the background with magnetic field in the SO(32) group is obtained by a T-duality from the right truncated model, the spectrum is the same as in (3.16) with $Q_L = 0$.[5] and Q_R being the momentum on the even self-dual lattice of the SO(32) gauge group. For convenience we write down the spectrum

$$M^2 = 2f(Q_R + \tilde{M})\left(l + \frac{1}{2}\right) - 2f(Q_R + \tilde{M})S_L + \frac{4}{\alpha'}\left(N_L - \frac{1}{2}\right). \tag{3.23}$$

As in the discussion of the right truncated model, states with $N_L = 1/2$, $Q_R = \sqrt{2}$ corresponds to off-diagonal fluctuations charged under the the background U(1). But unlike that case it is easy to obtain these states as fluctuations around the supergravity background as the low energy effective action contains the full SO(32) gauge fields including the off-diagonal fluctuations. There are tachyons with higher spins as in the right truncated model and these tachyons cannot be seen in supergravity. We will call this lowest lying tachyon as the Nielsen-Olesen tachyon.

4. The Nielsen-Olesen instability in supergravity

We have seen that to study the Nielsen-Olesen tachyon in supergravity we need to embed the uniform magnetic field in the SO(32) gauge group. In this section we discuss some important properties of this background and perform a linearized fluctuation analysis around the supergravity background and determine the tachyon explicitly.

To set up notations and conventions we write down the bosonic part of the low energy effective action of the heterotic string theory is given by

$$S = \frac{1}{(2\pi)^7(\alpha')^4}\int d^{10}x\sqrt{-g}e^{-2\Phi}\left[R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{8}\frac{\alpha'}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right]. \tag{4.1}$$

where R is the Ricci scalar, $F_{\mu\nu}$ denotes the non-abelian field strength. The gauge group is SO(32) with the gauge fields in the vector representation of SO(32). There is an extra factor of 1/2 in the action for the gauge field in the above equation as we have normalized the generators so that $\mathrm{Tr}(T^aT^b) = 2\delta^{ab}$. $H_{\mu\nu\rho}$ is the field strength associated with the $B_{\mu\nu}$ field

$$\begin{aligned} H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{4}\frac{\alpha'}{2}\,\mathrm{Tr}\left(A_\mu F_{\nu\rho} - \frac{i}{3}A_\mu[A_\nu, A_\rho]\right) + \\ + \text{cyclic permutations of } \mu, \nu, \rho\,. \end{aligned} \tag{4.2}$$

[4]The author thanks Ashoke Sen for pointing this out.

[5]We have decompactified the original S^1 we started out with.

Performing the T-duality in (3.19) on the solution in (3.2) we obtain

$$\begin{aligned} ds^2 &= -(dt + a_i dx^i)^2 + dx^i dx_i + dx^m dx_m \\ A_i &= \sqrt{2}\frac{\epsilon_{ij} f x^j}{2}\,, \qquad e^{2\Phi} = g_s^2\,, \qquad B_{ti} = a_i\,, \end{aligned} \tag{4.3}$$

where we have scaled the U(1) gauge field by $\sqrt{2/\alpha'}$ so that it has dimensions of mass. It is more convenient to perform calculations in the Landau gauge. We can perform the following gauge and co-ordinate transformation to convert the background to the Landau gauge

$$\begin{aligned} t &\to t + \frac{\tilde{f}}{2} x^1 x^2\,, \\ A_i &\to A_i + \partial_i \alpha\,, \qquad \alpha = \frac{f}{2} x^1 x^2\,. \end{aligned} \tag{4.4}$$

After the gauge transformation we obtain the following solution in the Landau gauge.

$$\begin{aligned} ds^2 &= -(dt + a dx^1)^2 + dx^i dx_i + dx^m dx_m \\ a &= \sqrt{\frac{\alpha'}{2}} f x^2\,, \qquad A_1 = Q f x^2 L_3 \\ B_{t1} &= \tilde{f} x^2\,, \qquad e^{2\Phi} = g_s \end{aligned} \tag{4.5}$$

The above metric is not asymptotically flat, in fact it has a constant curvature given by

$$R = \frac{1}{2}\tilde{f}^2\,, \qquad R_{\mu\nu}R^{\mu\nu} = \frac{3}{4}\tilde{f}^4 \tag{4.6}$$

Therefore for small field strength $\tilde{f}$ we can trust our analysis in supergravity and as the dilaton is constant, we can supress string loop corrections by tuning g_s to be small.

4.1 Fluctuations around a constant magnetic field in supergravity

In this section we determine how the Nielsen-Olesen tachyon occurs as a fluctuation of the fields in supergravity, we will explicitly show that the spectrum of fluctuation is given by (3.18). We have seen that the Nielsen-Olesen tachyon is an off diagonal fluctuation of the gauge field. Therefore consider the following fluctuations of the solution in (4.5) which involves only off diagonal components of the gauge field.

$$\begin{aligned} A_\mu &= \delta_{1\mu} f x^2 L_3 + W_\mu^+ L_+ + W_\mu^- L_- \\ g_{\mu\nu} &= g_{\mu\nu}^{(0)}\,, \qquad B_{\mu\nu} = B_{\mu\nu}^{(0)}\,, \qquad \Phi = \Phi^{(0)} \end{aligned} \tag{4.7}$$

here the superscript ${}^{(0)}$ refers to the background in (4.5). The equation of motion for these fluctuations reduce to

$$\hat{D}^\nu \hat{D}_\nu W_\mu - 2i[F^\nu{}_\mu, W_\nu] + R_\mu{}^\nu W_\nu = 0 \tag{4.8}$$

here W_μ stands for the off diagonal fluctuations W_μ^+ and W_μ^- and $\hat{D}_\mu$ refers to the covariant derivative given by $\hat{D}_\mu W_\nu = \partial_\mu W_\nu - \Gamma^\rho_{\mu\nu} W_\rho + i[A_\mu, W_\nu]$. In (4.8) we have imposed the background gauge condition

$$\hat{D}^\mu W_\mu = 0\,. \tag{4.9}$$

To solve the equations (4.8) and (4.9) for fluctuations it is convenient to use an ansatz. We will demonstrate it for the tachyons in the lowest Landau level. Consider the following off diagonal fluctuations

$$\begin{aligned} W_i^+ &= \left(\frac{f(Q+\tilde{M})}{4\pi l_1^2}\right)^{1/4} e^{-iMt} e^{-ik_1x^1} e^{-\frac{f}{2}(Q+\tilde{M})\left(\frac{k_1}{f(Q+\tilde{M})}-x^2\right)^2} \begin{pmatrix} 1 \\ i \end{pmatrix} \\ W_i^- &= \left(\frac{f(Q+\tilde{M})}{4\pi l_1^2}\right)^{1/4} e^{iMt} e^{-ik_1x^1} e^{-\frac{f}{2}(Q+\tilde{M})\left(\frac{k_1}{f(Q+\tilde{M})}+x^2\right)^2} \begin{pmatrix} 1 \\ -i \end{pmatrix} \\ \text{with} &\quad W_0^\pm = W_m^\pm = 0\,. \end{aligned} \tag{4.10}$$

Substituting the above ansatz in (4.8) and (4.9) one can show that the equations of motion and the background gauge conditions are satisfied if

$$M^2 = -f(Q+\tilde{M})\,. \tag{4.11}$$

This is the condition on the spectrum of for the tachyon in the lowest Landau level given in (3.18). The excited state in the $l=0$ Landau level is obtained by interchanging the two component vectors in (4.10)

$$\begin{pmatrix} 1 \\ i \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 \\ -i \end{pmatrix}. \tag{4.12}$$

Higher Landau levels are obtained by replacing the gaussian in (4.10) by the appropriate wave function of the harmonic oscillator. Thus the spectrum given in (3.18) is reproduced. Note that the tachyons are localized and for small values of the field strength f the spectrum reduces to that of that of off diagonal fluctuations in the SO(3) field theory found by Nielsen-Olesen. For later use we remark that the wave functions of the type constructed here with $M=0$ are eigen functions of the operator

$$O_{ij}^\pm = \hat{D}^k\hat{D}_k\delta_{ij} \mp f\epsilon_{ij} + R_{ij}\,, \tag{4.13}$$

with eigen values proportional to f.

5. Condensation of the Nielsen-Olesen tachyon in supergravity

In this section we argue that the Nielsen-Olesen tachyon in supergravity drives the background (4.5) to flat space. Our argument rests on the three evidences discussed below.

5.1 The S-dual in type-I theory

We have seen in the previous section that when the uniform magnetic field is embedded in the SO(32) gauge group of the heterotic string the off diagonal fluctuations which are charged with respect to the background U(1) contain tachyonic modes. In the heterotic string these are localized closed string modes in the twisted sector. S-duality of the heterotic string maps it to type-I string. From the mass formula in (3.23) we see that these modes are charged under the background U(1) in heterotic string, hence one would expect these modes to become open string modes in the type-I string. Therefore, it is possible to derive a decoupling limit in which the closed strings decouple. Then the tachyon condensation process reduces to that of tachyon condensation process in field theory discussed in section 2. The tachyon drives the system to the Yang-Mills vacuum. Thus we would expect by continuity, that turning on the type-I coupling the tachyon will drive the system to flat space. We now discuss this argument in detail.

Consider the S-dual of the constant magnetic field background in heterotic string with the magnetic field from the SO(32) gauge group given in (4.5). S-duality involves the field redefinitions given by.

$$\phi_{(I)} = -\Phi_{(H)}\,, \qquad ds^2_{(I)} = e^{-\Phi_H} ds^2_{(H)}\,, \qquad B^{(I)}_{ti} = B^{(H)}_{ti}\,, \qquad A^{(I)}_\mu = A^{(H)}_\mu\,. \tag{5.1}$$

Using this map we obtain the solution

$$\begin{aligned} ds^2 &= -(dt + b dx^1)^2 + dx^i dx_i + dx^m dx_m \\ b &= \sqrt{\frac{\alpha'}{2g_I}} f x^2\,, \qquad A_1 = Q\frac{1}{g_I} f x^2 L_3 \\ B^{RR}_{ti} &= \sqrt{\frac{\alpha'}{2g_I}} f x^2\,, \qquad e^{\Phi_I} = g_I = \frac{1}{g_s}\,. \end{aligned} \tag{5.2}$$

Here we have rescaled coordinates by $1/\sqrt{g_I}$ to get rid of the pre-factors in front of the metric. Under the duality map given in (5.1) the RR 2-form B^{RR} does not couple to the dilaton by the usual prefactor $e^{-2\Phi_I}$, to ensure that the RR form couples in the usual way we have scale the RR form by g_I so that it appears in the action with the usual dilaton coupling. This coupling of the RR form 2-form is the natural one if one derives the low energy effective action using scattering amplitudes in string theory [23]. The S-duality has converted the gauge field to the open string sector.

It is easy to see from the above background that the closed string fields is suppressed by a factor of $\sqrt{g_I}$ with respect to that of the open string gauge field. We can make use of this to derive the decoupling limit. This is essentially a limit in which the back reaction due to the presence of the magnetic flux which results in a change in the closed string fields, the metric and the Ramond-Ramond background is neglected. The limit is given by

$$g_I \to 0\,, \qquad \text{with } \frac{f}{g_I} = h \text{ held fixed} \tag{5.3}$$

Note that the decoupling limit in type I corresponds to infinite coupling in the heterotic side. In this limit the background in (5.2) reduces to a flat metric with the Ramond-Ramond flux set to zero. The gauge field is given by

$$A_1 = Qhx^2 L_3\,. \tag{5.4}$$

The type-I open string in this constant gauge field can be quantized [24]–[27]. From the spectrum it can be seen that there exists a Nielsen-Olesen tachyon which survives in the $\alpha' \to 0$ limit. The tachyon condensation process can now be studied in field theory and from the discussion in section 2 we can conclude that it drives the system to the vacuum. On the heterotic side this implies that at strong coupling the tachyon will drive the system to flat space. It is natural to expect by continuity that lowering the coupling to weak coupling on the heterotic the result would not change. There could be a possible phase transition which might affect this conclusion therefore we will discuss the the tachyon condensation process directly in the heterotic theory in the next subsections.

As an aside we remark that since the tachyons in the background (5.2) are in the open string sector, they can be studied using Berkovits' open super string field theory.

5.2 The closed string tachyon potential

In this section we obtain the closed string tachyon potential for the background with uniform magnetic field in the SO(32) group of the heterotic string theory given in (4.3) in supergravity. We show that the minimum of the tachyon potential is flat space. To obtain the tachyon potential we have to define a notion of energy for the space (4.3), as this space is not asymptotically flat it is difficult to define a notion of energy. However, the metric in (4.3) admits a killing vector $\xi^\mu = (1, 0, 0)$, (we have listed only the relevant co-ordinates). This can be used to show that the following integral of the stress energy tensor is a conserved quantitiy

$$E = \frac{1}{(2\pi)^7(\alpha')^4 g_s^2} \int d^9x \sqrt{h} T_{\mu\nu}\eta^\mu \xi^\nu \,, \tag{5.5}$$

here η^μ is the unit normal to a space like surface Σ_t of constant t and h_{ij} is the induced metric on Σ_t. All fluctuations of the metric should preserve the constraint equations of gravity, as these are constraints for intial velocities. Therefore, to obtain the tachyon potential we evaluate the energy functional given in (5.5) on the fluctuations which are subject to the gravity constraints.[6]

We first restrict our fluctuations to preserve the form of the metric, and the B-field of the background given by

$$\begin{aligned} &ds^2 = -(dt - g_i(x^1, x^2)dx^i)^2 + dx^i dx_i + dx^m dx_m \\ &B_{ti} = b_i(x^1, x^2) \qquad e^\Phi = g_s \qquad A_i = A_i(x^1, x^2) \end{aligned} \tag{5.6}$$

We have allowed dependence of the functions only on the x^1, x^2 directions as the zero momentum tachyon ($k^m = 0$), depends only on these coordinates. Since the background and the tachyons do not involve the dilaton we have assumed it to be constant. We also assume that the background and the fluctuations are small, so that deviations from flat space will be small. Now we use the constraints from the supergravity equations to eliminate the metric, and the B field in terms of the gauge field. The constraint is given by the 00 component of the Einstein equation.

$$R_{00} - \frac{1}{2}g_{00}R = \frac{1}{4}H_{0\mu\nu}H_0{}^{\mu\nu} - \frac{1}{2}g_{00}\left(\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{\alpha'}{16}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\right). \tag{5.7}$$

[6]This method is inspired by discussions with M. Gutperle, M. Headrick and S. Minwalla during the collaboration [9].

Here we have set the dilaton to be constant. The constraint can be satisfied if $g_i = b_i$ and by requiring[7]

$$-(\partial_1 g_2 - \partial_2 g_1)^2 + \frac{\alpha'}{8}\,\mathrm{Tr}(F_{12}F_{12}) = 0\,. \tag{5.8}$$

Thus the only degree of freedom left over is that of the gauge field. Using these constraints in the energy formula in (5.5) and retaining only the leading order term in the fields we obtain[8]

$$\begin{aligned} E &= \frac{1}{(2\pi)^7(\alpha')^4 g_s^2}\int d^9x \left(R_{00} - \frac{g_{00}}{2}R\right) \\ &= \frac{1}{8(2\pi)^7(\alpha')^3 g_s^2}\int d^9x\,\mathrm{Tr}(F_{12}F_{12})\,. \end{aligned} \tag{5.9}$$

Tachyon condensation should proceed by minimizing this energy. The energy functional is proportional to the negative of the Yang-Mills action. Thus the calculation of the tachyon potential reduces to that performed in section 2. Considering fluctuations involving only the tachyon and integrating out the diagonal fluctuation ϕ we obtain the following potential

$$E = \frac{1}{8(2\pi)^7(\alpha')^4 g_s^2}\int d^8x \left(\frac{4\pi}{\sqrt{l_1 l_2}} - |\chi|^2\right)^2 \tag{5.10}$$

here we have chosen the background field strength $f = 2\pi/l_1 l_2$, considering a single unit of flux on the $1-2$ plane and regulated the system in a large box of size l_1 and l_2. The minimum of the tachyon potential cancels the energy due to the background magnetic field. Though our analysis above was restricted only to fluctuations of the gauge field in the lowest Landau level we have seen in section 2, that even on the inclusion of the higher levels the potential is minimized at $F_{12} = 0$. From the constraint (5.8) this implies that the curvature which is proportional $(\partial_1 g_2 - \partial_2 g_1)^2$ vanishes at $F_{12} = 0$. Thus the Nielsen-Olesen tachyon in supergravity triggers the decay to flat space.

It is clear from the orbifold examples in [3], that energy is a good observable to compare in closed string tachyon condensation. The orbifold examples considered there show that the localized closed string tachyons condense so that energy is minimized. This example also falls into this simple pattern. The fact that this model admits a tachyon potential with a fixed point ensures that it is possible to study time dependent evolution of the tachyon. During the condensation process the space changes from a space of constant curvature to flat space, a time dependent study of this phenomenon would be interesting.

5.3 Renormalization group flow in supergravity

In this section we study the RG flow induced by the tachyon in supergravity and show that the Nielsen-Olesen tachyon triggers the system to flow in the direction of decreasing field

[7] This is in fact the dilaton equation of motion.

[8] For the background in (5.6) the Einstein frame is the same as the string frame since the dilaton is constant.

strengths towards flat space. The RG equations are given by

$$\begin{aligned}
\frac{\dot{g_{\mu\nu}}}{\alpha'} &= -R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \Phi + \frac{1}{4} H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} + \frac{\alpha'}{8} \operatorname{Tr}(F_{\mu\alpha} F_\nu^\alpha)\,, \\
\frac{\dot{\Phi}}{\alpha'} &= -\frac{1}{2} \nabla^\mu \nabla_\mu \Phi + \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{\alpha'}{64} \operatorname{Tr}(F_{\mu\nu} F^{\mu\nu})\,, \\
\frac{\dot{B}_{\mu\nu}}{\alpha'} &= \frac{1}{2} \nabla^\alpha H_{\alpha\mu\nu} - \partial^\alpha \Phi H_{\alpha\mu\nu}\,, \\
\frac{\dot{A}_\mu}{\alpha'} &= \frac{1}{2} \hat{D}^\alpha F_{\alpha\mu} - \partial^\alpha \Phi F_{\alpha\mu} + \frac{1}{4} F_{\nu\rho} H_\mu{}^{\nu\rho}\,.
\end{aligned} \tag{5.11}$$

The renormalization group equations are complicated, however we can obtain a consistent set of RG equations for backgrounds containing small magnetic fields. We make the following ansatz for the solutions of the RG flow which is valid to order $O(\epsilon^2)$

$$\begin{aligned}
ds^2 &= -(dt + a_i(x^1, x^2)dx^i)^2 + dx^i dx_i + dx^m dx_m \\
a_i &= \epsilon_{ij} \tilde{f} x^j + g_i(x^1, x^2)\,, \qquad B_{ti} = \epsilon_{ij} \tilde{f} x^j + b_i(x^1, x^2) \\
A_i^3 &= \epsilon_{ij} Q f x^j + \phi_i(x^1, x^2)\,, \qquad A_i^{\pm} = W_i^{\pm}(x^1, x^2)\,.
\end{aligned} \tag{5.12}$$

Here ϕ_i, g_i are of the $O(\epsilon^2)$ and the off diagonal fluctuations of the gauge bosons which contain the tachyon occur at the $O(\epsilon)$. We assume that the background field f to be small and $f \sim O(\epsilon^2)$. Using this ansatz we show that we will obtain a consistent set of RG equations up to $O(\epsilon^3)$. The off diagonal fluctuations are expanded in the complete set of functions which satisfy the background gauge condition $\hat{D}^i W_i^{\pm} = 0$ and are eigen functions of the operator $O_{ij}^{\pm}$ given in (4.13). The functions ϕ_i also satisfy the background gauge condition $\hat{D}^i \phi_i = 0$. In section 4.1 we have constructed these eigen functions. An important property of these functions is that $F_{ij}^{\pm} \sim O(\epsilon^3)$. This can be verified for the lowest Landau wave functions given in (2.13), and it is easy to see that it holds for the higher Landau levels.

Whatever initial conditions we start our RG evolution, they must satisfy the constraint equations of gravity given in (5.7) since these are constraints on the initial velocities. As discussed in section 5.2 the constraint equations can be satisfied by imposing $g_i = b_i$ and (5.8). If these equations are satisfied the 00 component of the RG equation for the metric and the RG equation for the dilaton are automatically satisfied. The remaining RG equations for the metric reduce to

$$\dot{g}_i = \frac{\alpha'}{2} \partial_j G_{ji}\,, \tag{5.13}$$

where we have used $g_i = b_i$ and (5.8). For the field strength $G_{12} = -\tilde{f} + \partial_1 g_2 - \partial_2 g_1$ the above equation implies

$$\dot{G}_{12} = \frac{\alpha'}{2} (\partial_1^2 + \partial_2^2) G_{12}\,. \tag{5.14}$$

The 01 and the 02 RG equations for the B-field also reduce to the above set of equations while the 12 equations is satisfied to $O(\epsilon^3)$. The RG equations for the gauge field to $O(\epsilon^3)$

are given by

$$
\begin{aligned}
\dot{\phi}_i &= \frac{\alpha'}{2}\partial_j F^3_{ji} \\
\dot{W}^+_i &= \frac{\alpha'}{2}\Big[O^+_{ij}W^+_j - iW^+_j\partial_j\phi_i + i\phi_j\partial_j W^+_i - iW^+_j(\partial_j\phi_i - \partial_i\phi_j) + \\
&\quad + 2W^+_j(W^+_j W^-_i - W^-_i W^+_j)\Big] \\
\dot{W}^-_i &= \frac{\alpha'}{2}\Big[O^-_{ij}W^+_j + iW^-_j\partial_j\phi_i - i\phi_j\partial_j W^-_i + iW^-_j(\partial_j\phi_i - \partial_i\phi_j) + \\
&\quad + 2W^-_j(W^-_j W^+_i - W^+_i W^-_j)\Big]\,.
\end{aligned} \tag{5.15}
$$

Here $F^3_{ij} = -Qf\epsilon_{ij} + \partial_i\phi_j - \partial_j\phi_i + 2i(W^+_i W^-_j - W^-_j W^+_j)$. The RG equation for the dilaton relates the metric with the gauge field, to $O(\epsilon^3)$ this constraint is given by

$$
G_{12} = \sqrt{\frac{\alpha'}{2}}\frac{1}{Q}F^3_{12}\,. \tag{5.16}
$$

From the RG equation for the diagonal component of the gauge field we obtain the following equation

$$
\dot{F}^3_{12} = \frac{\alpha'}{2}(\partial^2_1 + \partial^2_1)F^3_{12}\,. \tag{5.17}
$$

Thus we see that the RG equation for the field strength constructed from the metric (5.14) and the RG equation for the diagonal component of the gauge field strength are consistent to $O(\epsilon^3)$. The study of the RG flow has now reduced to the flows of the gauge field and the above set of equations can be studied numerically by expanding $W^\pm_i$ in the eigen functions of the operator $O^\pm_{ij}$.

However, it is possible to show that there is a fixed point with vanishing field strength and curvature. The method of obtaining this solution is identical to that done just for the gauge field in the section 2.2. There we saw that the equations for the gauge field has a fixed point at $\phi_i = 0$ with all the coefficients of the Landau levels in $W^\pm_i$ vanishing except the tachyon. The RG equations for W^+_1 reduces to

$$
\dot{W}^+_1 = \frac{\alpha'}{2}\left(QfW^+_1 - 4W_1|W^+_1|^2\right)\,. \tag{5.18}
$$

Thus at the fixed point the tachyon gets an expectation value $|W^+_1|^2 = Qf/4$, the other components of the tachyon can be obtained from the relations in (2.27). This point corresponds vanishing of the field strengths $F^\pm_{12}$ and F^3_{12}. From (5.16) we see that G_{12} also vanishes. As the curvature of this space is proportional to G^2_{12}, we see that the RG flow set up by the tachyon ends up in flat space. It is easy to see from the RG equations for the field strength that expectation value for the tachyon drives the RG flow to decreasing curvatures towards flat space.

To study renormalization group flow induced by localized tachyons on orbifolds, an entropy corresponding to localized closed string states called g_{cl} was defined in [5], and it was conjectured that g_{cl} decreases along the RG flow. It has been shown in [28] that the gauge degrees of freedom do not contribute to g_{cl}. In the system with magnetic field from the SO(32) group of the heterotic string the localized tachyons belong to the gauge degrees of freedom. Therefore at least these states are not taken into account by g_{cl}. It would be interesting to compute g_{cl} for the case with the magnetic field from the SO(32) gauge group and check if it decreases along the RG flow set up by the tachyon. Another physically motivated quantity which can decrease along RG flow is the energy. In all the orbifold examples of [3], the energy of the intial orifold is always higher than that of the end point. For the system studied in this paper it is clear that the energy functional given in (5.9) decreases along the RG flow as field strengths decrease along RG flow.

6. Stability against pair production

Constant fields in a theory are usually unstable to decay via pair production of particles which are are charged with respect to that field. For instance, a constant electric field in U(1) gauge theory decays by pair production of electron-positron pairs [16]. One might expect the backgrounds studied in this paper also might be unstable to decay via pair production of particles. In this section we show that these backgrounds are stable with respect to decay by particle creation. The reason can be traced to the fact that there is no particle creation in a electromagnetic plane wave [16]. All the invariants constructed from the electromagnetic field strength of the plane wave vanish and thus the effective action describing the vacuum polarization vanish. A similar argument was used in [29] to argue that there is no particle creation in a gravitational wave. We use the same method to show that there is no particle creation in the uniform magnetic field backgrounds considered in this paper.

It is convenient to first study particle creation in the background given in (3.1). The effective action describing vacuum polarization in this background is obtained by integrating virtual particles charged with respect to the background fields. Whatever the character of the virtual particles involved the effective lagrangian will be an invariant constructed out of the curvature tensor $R_{\mu\nu\rho\sigma}$ and the field strength $H_{\mu\nu\rho}$ and their derivatives.

The effective lagrangian contains terms of the type

$$\begin{aligned} L \sim\ & R\cdots \nabla R\cdots \nabla^2 R\cdots + H\cdots \nabla H\cdots \nabla^2 H\cdots + \\ & + R\cdots H\nabla R\cdots \nabla^2 R\cdots \nabla H\cdots \nabla^2 H\cdots . \end{aligned} \tag{6.1}$$

From the properties of the plane wave metric (3.1) given in appendix B. it is easy to see that the covariant indices of all quantities, the curvature, the metric, the connection and the field strength H contain only the indices u and i. While the contraviant indices of all quantities contain only the indices v, i. Thus any invariant constructed out of these quantities has to vanish. Note that terms which involve covariant derivatives of H and the curvature also have covariant indices u and i and contravariant indices v and i, therefore any invariant constructed out of covariant derivatives also vanish. Thus the effective lagrangian describing vacuum polarization vanishes and there is no pair production in these backgrounds. Due to this reason the effective action describing vacuum polarization for the background (3.2) which is obtained from the plane wave type background (3.1) by Kaluza-Klein reduction also vanishes. In summary we see that there is no pair production of particles in the backgrounds studied in this paper. It will be interesting to check this conclusion by performing an explicit 1-loop calculation in string theory analogous to [30]. The amplitude for pair production can be extracted from the imaginary part of the 1-loop amplitude.

7. Conclusions

We have studied tachyon condensation in backgrounds containing uniform magnetic fields in heterotic string theory. When the magnetic field is embedded in the SO(32) group of the the heterotic string it is possible to study the tachyonic mode corresponding to the Nielsen-Olesen instability within supergravity. The tachyon can be identified as a fluctuation of the supergravity fields. We constructed an energy functional and evaluated the closed string tachyon potential, the minimum of the tachyon potential corresponds to flat space. We studied the world sheet renormalization group flow in the supergravity approximation. For small values of the magnetic field we have obtained up a consistent set of renormalization group equations. We showed that the RG flow set up by the tachyon drives the background to flat space. This implies that tachyon signals the instability of the uniform magnetic field background to decay to flat space. Thus this system with localized closed string tachyon falls into the general pattern that, localized closed string tachyons tend to decay to flat space.

This system is of further interest as it admits a dual description in type-I theory. In type-I theory the tachyons studied in the heterotic string are in the open string sector. In open string tachyon condensation the boundary entropy or the tension of the branes involved is a quantity which decreases along RG flow set up by the tachyon. It would be interesting to find out what the boundary entropy would be for the type-I background considered in this paper. It is natural to expect that boundary entropy on the type-I side will be given by the tachyon potential evaluated in this paper.

Finally, most systems which are unstable due to perturbative tachyons are also unstable non-perturbatively due to tunneling and the end points are usually same For example the D-brane anti D-brane systems are unstable to formation of a bounce eating up the branes [31, 32] and in the Melvin background the perturbative process of tachyon condensation and the non-perturbative process involving brane nucleation leads to the same end point [33]. On the other hand the systems studied in this paper are stable with respect to decay by pair production particles. Essentially this is because these systems are Kaluza-Klein reduction of a special classes of plane waves.

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A. The tachyon potential for the Nielsen-Olesen instability

In this appendix we provide the details in evaluating the tachyon potential. We substitute the expansion given in (2.21) into the Yang-Mills action and obtain an effective action for the tachyon. We will organize the terms in the potential according to the order in fluctuations. The quadratic term in the fluctuation is given by

$$-\frac{1}{4g_{\mathrm{YM}}^2}\int d^4x \,\mathrm{Tr}\left(-2\delta A_\nu\left(D_\mu D^\mu \delta A^\nu - 2i[F^{\mu\nu},\delta A_\mu]\right)\right). \tag{A.1}$$

Substituting the fluctuation in (2.21) in (A.1) we obtain the following quadratic terms

$$S_2 = -\frac{1}{4g_{\mathrm{YM}}^2}\int d^2x\left(-16\sqrt{l_1 l_2}fQ|\chi|^2 + 4\int d^2k\frac{l_1l_2}{4\pi^2}\phi(-k)k^2\phi(k)\right) \tag{A.2}$$

here the integral is over x^0 and x^3, we have performed the integral over x^1 and x^2. The cubic term in fluctuations is given by

$$-\frac{1}{4g_{\mathrm{YM}}^2}\int d^4x \,\mathrm{Tr}\left(2i[\delta A_\mu,\delta_\nu]\{D^\mu\delta A^\nu - D^\nu\delta A^\mu\}\right). \tag{A.3}$$

Again substituting the fluctuation from (2.21) in the above equation we obtain

$$S_3 = \frac{16}{4g_{\mathrm{YM}}^2}\int d^2x\int d^2k\left(\frac{l_1l_2}{4\pi^2}|k|\phi(k)e^{-\frac{k^2}{2f}}|\chi|^2\right). \tag{A.4}$$

Finally, the quartic term in the fluctuation is given by

$$-\frac{1}{4g_{\mathrm{YM}}^2}\int d^4x - \mathrm{Tr}\left([\delta A_\mu,\delta A_\nu][\delta A^\mu,\delta A^\nu]\right). \tag{A.5}$$

Substituting (2.21) in (A.5) we get

$$S_4 = 16\frac{l_1l_2fQ}{\pi}|\chi|^4. \tag{A.6}$$

The total action in these fluctuations is given by $S = S_2 + S_3 + S_4$. We can eliminate the $\phi(k)$ using its classical equation of motion. This gives the following

$$\phi(-k) = 2\frac{e^{-\frac{k^2}{2f}}}{|k|}|\chi|^2. \tag{A.7}$$

Now substituting this value of ϕ in the action we obtain the following effective action for the tachyon

$$S = -\frac{16}{4g_{\mathrm{YM}}^2}\int d^2x - fQ\sqrt{l_1l_2}|\chi|^2 + \frac{fQl_1l_2}{2\pi}|\chi|^4. \tag{A.8}$$

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B. Properties of the plane wave metric

We list useful properties of the plane wave metric given in (3.1) The metric and inverse metric components are given by

$$\begin{aligned} g_{uv} &= \frac{1}{2}\,, \qquad g_{iu} = \frac{a_i}{2}\,, \qquad g_{ij} = \delta ij\,, \\ g^{uv} &= 2\,, \qquad g^{vv} = a_1^2 + a_2^2\,, \qquad g^{vi} = -a_i\,, \qquad g^{ij} = \delta^{ij}\,. \end{aligned} \tag{B.1}$$

The Cristoffel symbols are given by

$$\Gamma^i_{uj} = \frac{1}{4}\epsilon_{ij}\tilde{f}\,, \qquad \Gamma^v_{ui} = \frac{1}{4}\epsilon_{ij}a_j\tilde{f}\,. \tag{B.2}$$

Finally the components of the curvature tensor and the B-field are given by

$$R_{iuju} = -\delta_{ij}\frac{\tilde{f}^2}{16}, \quad H_{jiu} = \epsilon_{ij}\tilde{f}\,. \tag{B.3}$$

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